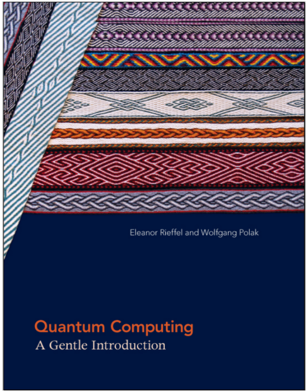
PHYS 407 - Introduction to Quantum Computing

Term: Spring 2022
Meetings: Tuesday & Thursday 17:00-18:15
Location: Room 240 Pritzker Science
Video: All sessions recorded for online viewing

Instructor: Carlo Segre
Office: 166D/172 Pritzker Science
Phone: 312.567.3498
email: segre@iit.edu

Book: *Quantum Computing: A Gentle Introduction*, E. Rieffel & W. Polak (MIT Univ Press, 2011)

Web Site: http://phys.iit.edu/∼segre/phys407/22S



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5. Use the concept of quantum entanglement to develop quantum algorithms.
6. Clearly describe the techniques of quantum error correction and fault tolerance.
7. Build quantum algorithms using IBM Qiskit.

Course grading (tentative!)

30% – Homework assignments

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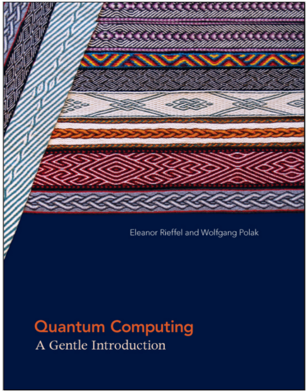
Grading scale

A	–	88%	to	100%
B	–	75%	to	88%
C	–	62%	to	75%
D	–	50%	to	62%
E	–	0%	to	50%

Topics to be covered (chapter titles)

1. Quantum building blocks



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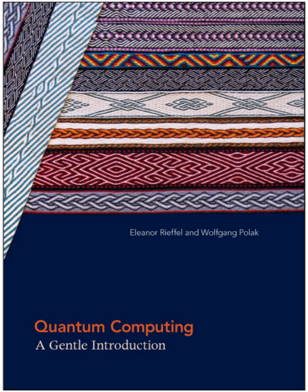

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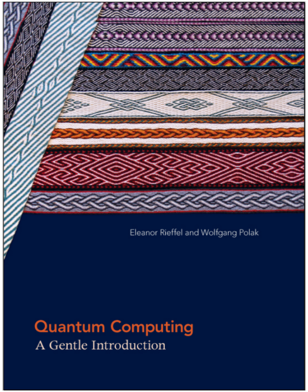

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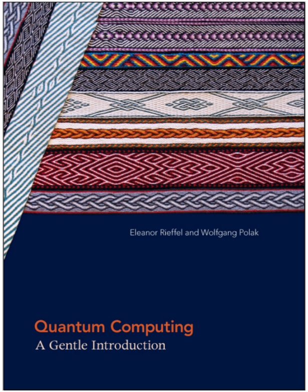

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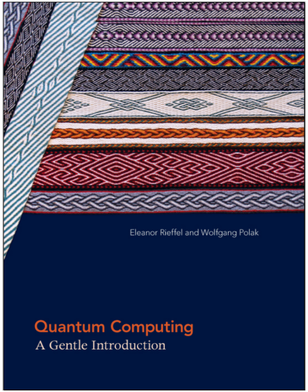
5. Other topics as appropriate



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Quantum computing is becoming interesting to a number of fields outside physics and could be even more relevant in the near future

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MRCAT

Current active membership

Illinois Tech

Argonne Chem. Sci. & Eng.

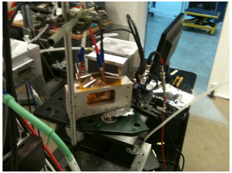
Argonne Biosciences

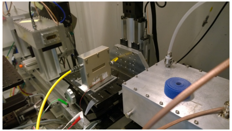
EPA Cincinnati

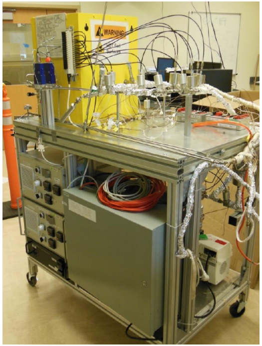
UOP Honeywell

BP p.l.c.

- Materials Research Collaborative Access Team
- Specializing in x-ray absorption spectroscopy for local structure & electronic measurements
- Focus on *in situ* experiments at time scales from 10 s to 2 min







My research in fuel cell catalysts. . .

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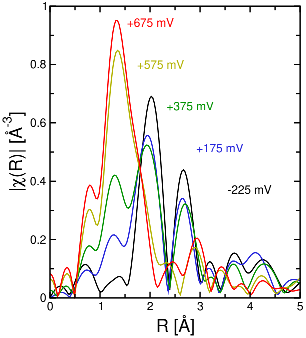

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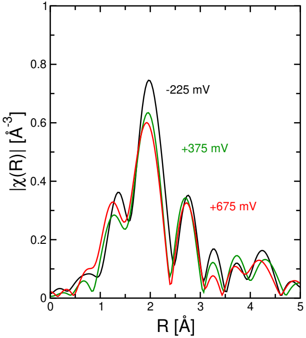

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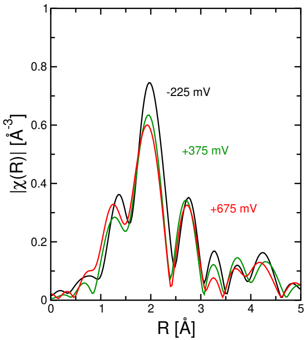

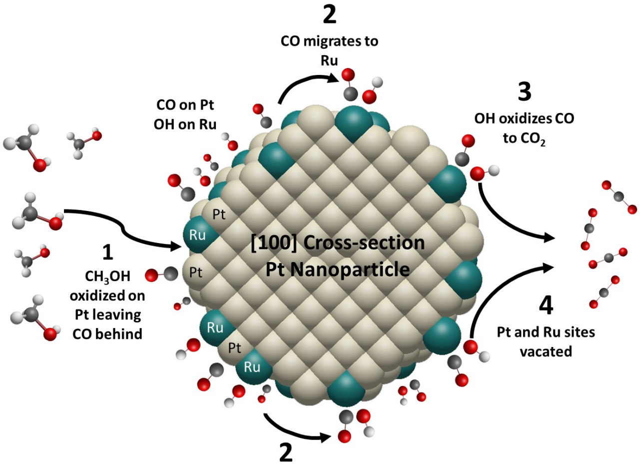

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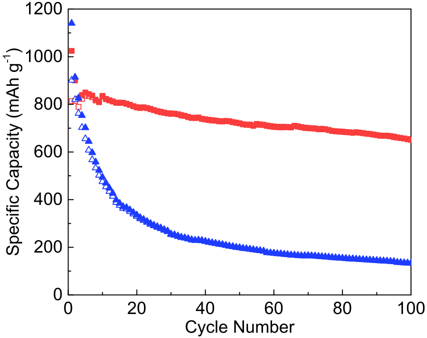

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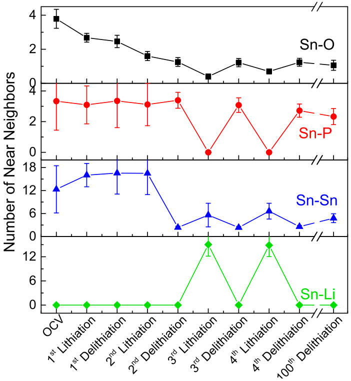

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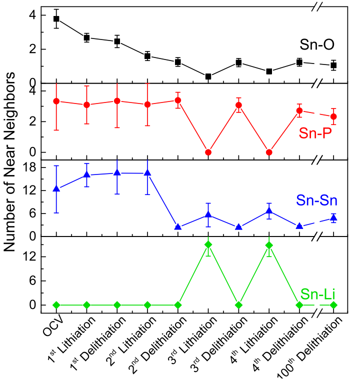

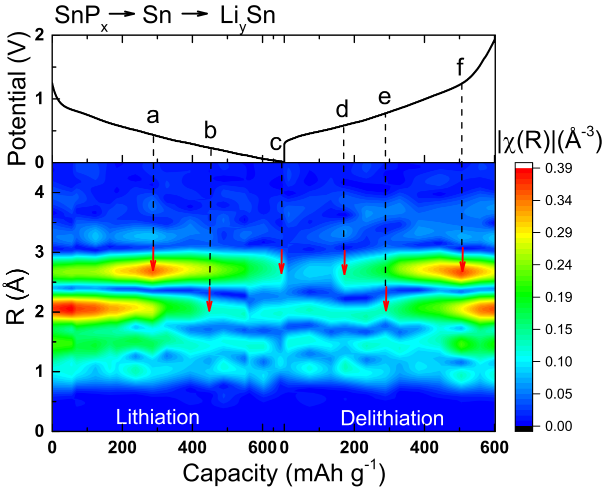

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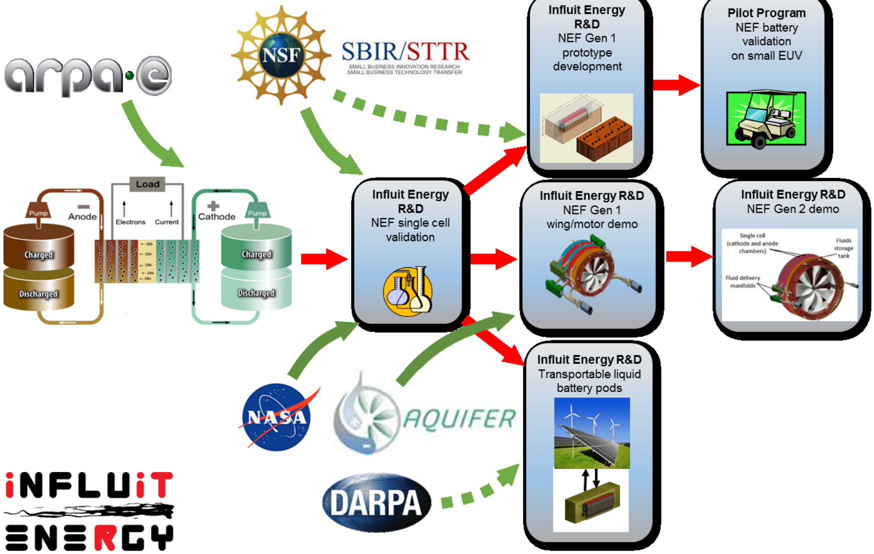

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Reading Assignment: Chapter 2.4-2.5; Chapter 3.1

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Homework Assignment #01:
Chapter 2:1,2,3,5,6,11
due Thursday, January 20, 2022

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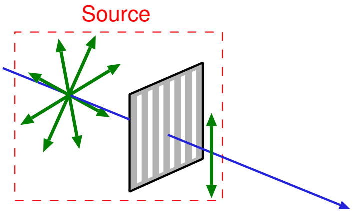

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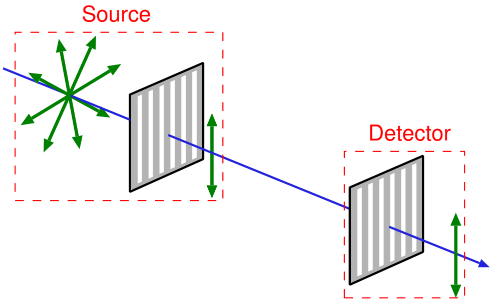

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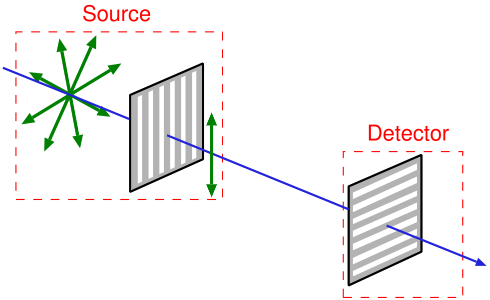
- a detector for the vertical state will detect the full beam intensity, a detector for the horizontal state will detect nothing



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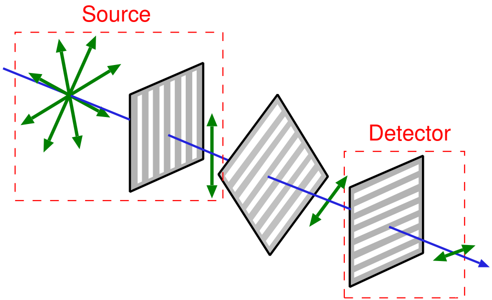

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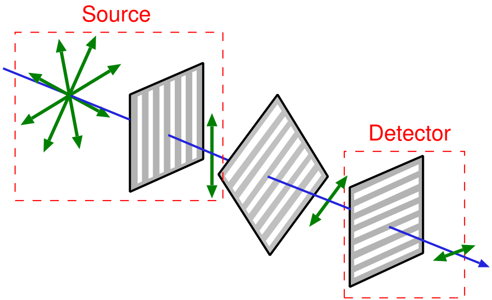

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because photons are quantum particles, this effect works even for single photons with the measuring a fraction of the photons to be horizontal

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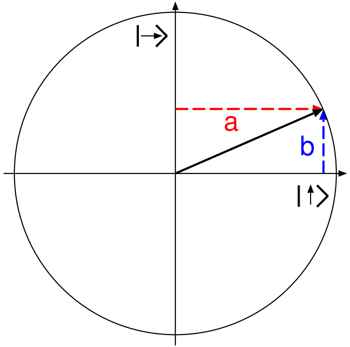

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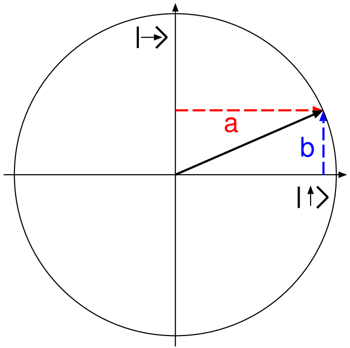

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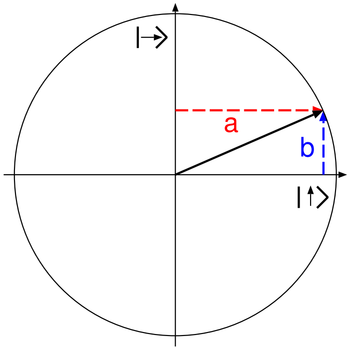

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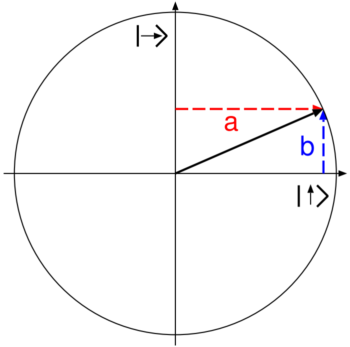

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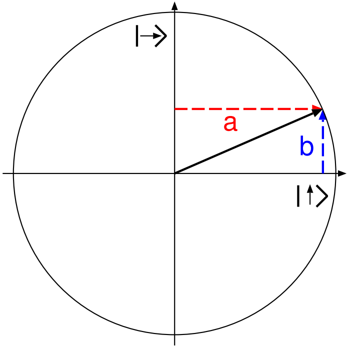

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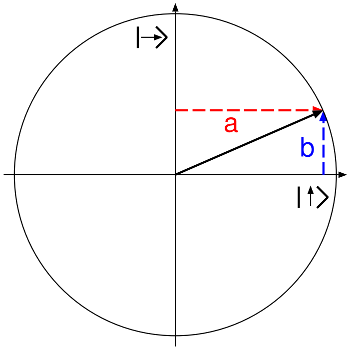

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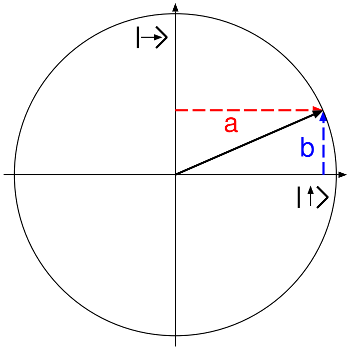
where the amplitudes a and b are complex constants such that the state is normalized



$$|v\rangle = a|\uparrow\rangle + b|\rightarrow\rangle,$$

$$|v|^2 \equiv 1 \quad \longrightarrow \quad |a|^2 + |b|^2 = 1$$

$$a = |a|e^{i\alpha}, \quad b = |b|e^{i\beta}$$

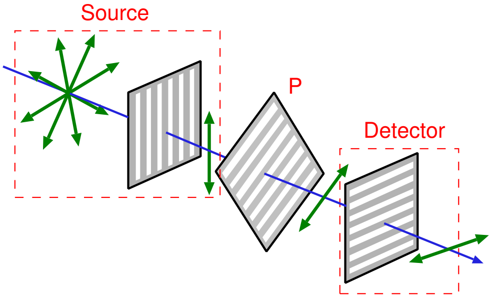
suppose a photon in a general state $|v\rangle$ enters a detector whose direction is $|\uparrow\rangle$

the probability of detection is $|a|^2$ and the probability of absorption is $|b|^2$

this formalism allows us to describe the polarization experiment

Polarizer experiment

The photons that come from the source are in a state $|\uparrow\rangle$



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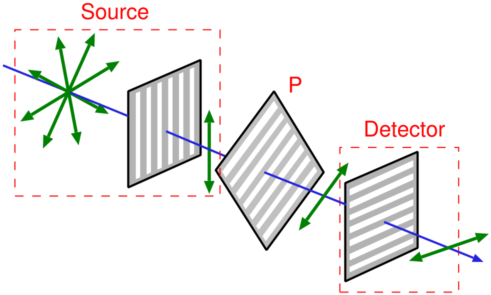

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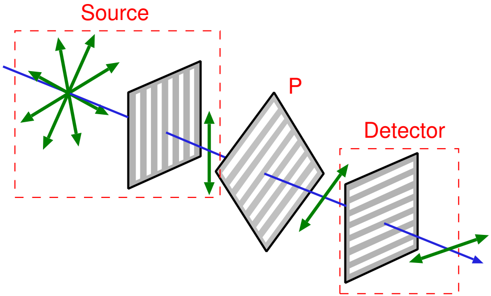

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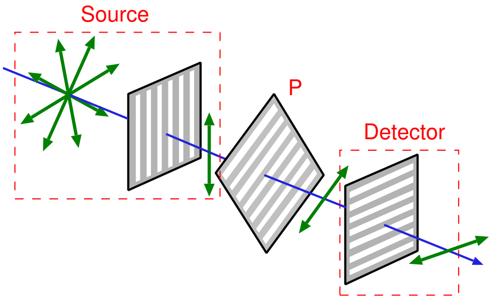
the photon thus has an 0.5 probability of passing through the polarizer and will then be in a state



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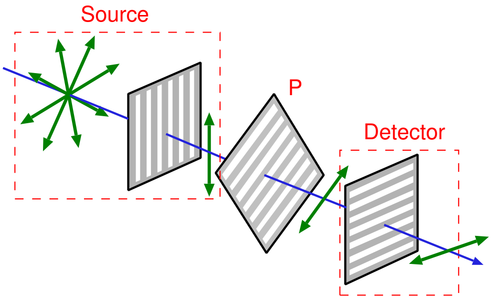

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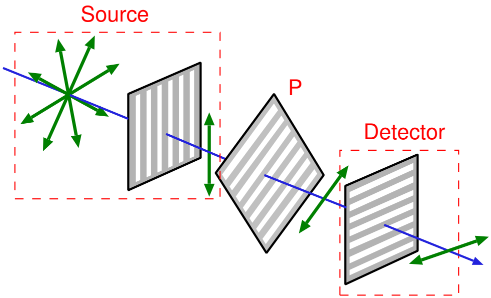

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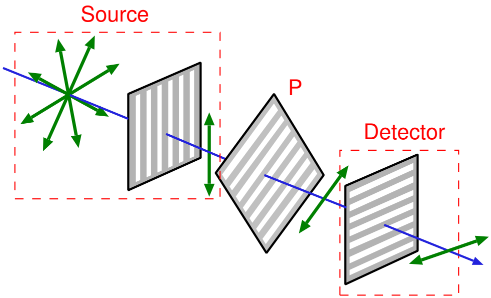

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quantum particles (and qubits) behave probabilistically

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In the standard basis, $\{|0\rangle, |1\rangle\}$, the vector $|v\rangle = a|0\rangle + b|1\rangle$ is

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

Similarity to linear algebra

The ket, $|\alpha\rangle$, corresponds to a column vector, α, in linear algebra while a bra $\langle\alpha|$ is its conjugate transpose, $\alpha^\dagger$, a row vector

$$|\alpha\rangle = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad \langle\alpha| = (\overline{a_1} \ \cdots \ \overline{a_n})$$

The inner product of two vectors is

$$\langle\alpha|\beta\rangle = (\overline{a_1} \ \cdots \ \overline{a_n}) \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^{n} \overline{a_i} b_i$$

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