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- First order perturbation theory review

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- Treating a two-fold degeneracy

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- Higher order degeneracies

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What can we do about degenerate states where the energy denominator vanishes?

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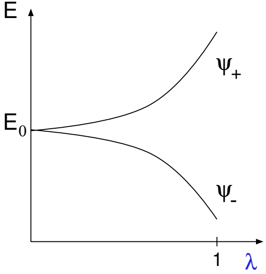

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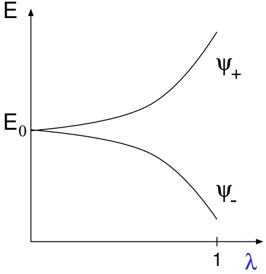

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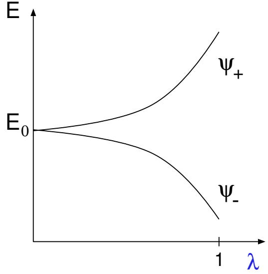

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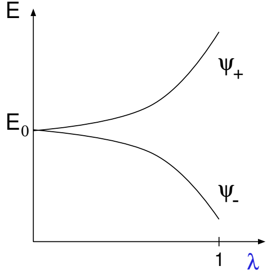

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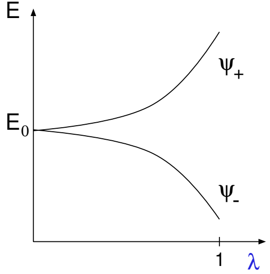

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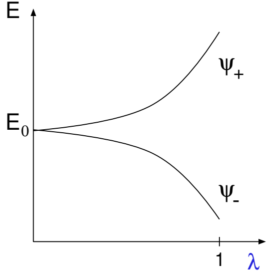

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Determining the energies

Solve these two equations for the energies of the two "ideal" orthogonal states

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$$\begin{aligned} 0 &= (E^1)^2 - E^1(W_{aa} + W_{bb}) + (W_{aa}W_{bb} - W_{ab}W_{ba}) \\ E^1_{\pm} &= \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} + W_{bb})^2 - 4(W_{aa}W_{bb} - W_{ab}W_{ba})}\right] \\ &= \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right] \end{aligned}$$

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It is often possible to choose the "ideal" linear combination through use of a convenient theorem

Determining α and β

Theorem: If A is a hermitian operator which commutes with both H^0 and H' and ψ_a^0 and ψ_b^0 are also eigenfunctions of A with distinct eigenvalues

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Determining α and β

Theorem: If A is a hermitian operator which commutes with both H^0 and H' and ψ_a^0 and ψ_b^0 are also eigenfunctions of A with distinct eigenvalues

$$A\psi_a^0 = \mu\psi_a^0, \quad A\psi_b^0 = \nu\psi_b^0, \quad \mu \neq \nu$$

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