Today's Outline - February 04, 2020

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- Fine structure

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Homework Assignment #04:
Chapter 7:8,12,13,17,33,36
due Tuesday, February 11, 2020

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- Fine structure
- Zeeman Effect
- Hyperfine splitting
- Problems from Chapter 7

Homework Assignment #04:
Chapter 7:8,12,13,17,33,36
due Tuesday, February 11, 2020

Homework Assignment #05:
Chapter 7:20,21,24,28,29,37
due Tuesday, February 18, 2020

Spin orbit interaction review

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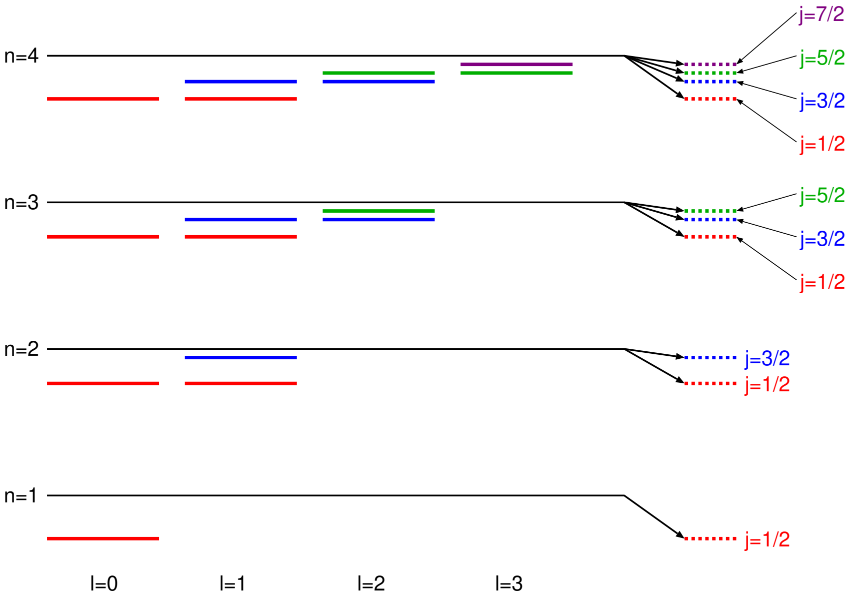
Fine structure of hydrogen



Sample calculations

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depending on the regime, we can use different kinds of perturbation theory

Problem 7.23

Consider the following relationship between angular momentum and magnetic field:

$$\vec{B} = \frac{1}{4\pi\epsilon_0}\frac{e}{mc^2 r^3}\vec{L}$$

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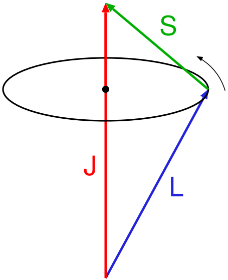

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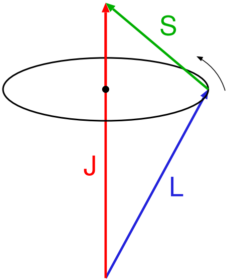

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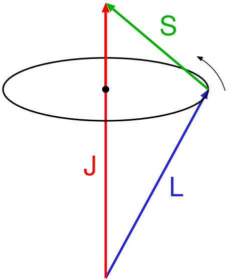

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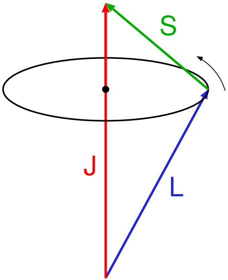

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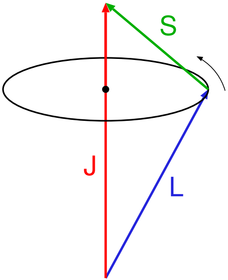

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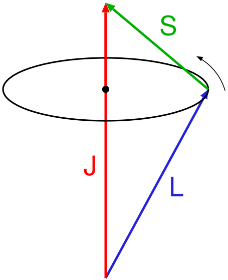

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Strong-field Zeeman effect

When $B_{ext} \gg B_{int}$, the spin-orbit coupling must be treated as the perturbation and the solutions must be eigenfunctions of the unperturbed wave functions with good quantum numbers: n, l, m_l, s, m_s.

If B_{ext} is in the $\hat{z}$ direction, the Zeeman Hamiltonian is

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$$E^1_{fs} = \frac{13.6\text{eV}}{n^3}\alpha^2 \left\{ \frac{3}{4n} - \left[\frac{l(l+1) - m_l m_s}{l(l+1/2)(l+1)} \right] \right\}, \quad l > 0$$

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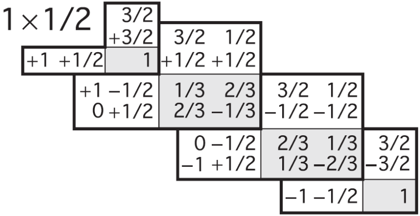
for $l = 1$, there are many more mixing possibilities since $m_l = -1, 0, 1$

it is convenient to use the $1 \times \frac{1}{2}$ Clebsch-Gordan table to generate all the possible states of $|j\, m_j\rangle$

Enumerating the $l = 1$ degenerate states



Pick out all the possible combinations to build the remaining degenerate states

$1 \times 1/2$

m_1	m_2	$J=3/2$, $M=+3/2$	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$	$J=3/2$, $M=-3/2$
+1	+1/2	1					
+1	−1/2		1/3	2/3			
0	+1/2		2/3	−1/3			
0	−1/2				2/3	1/3	
−1	+1/2				1/3	−2/3	
−1	−1/2						1

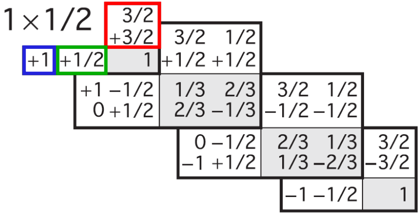
Enumerating the $l = 1$ degenerate states

Pick out all the possible combinations to build the remaining degenerate states

$1 \times 1/2$	3/2 +3/2	3/2 +1/2	1/2 +1/2	3/2 −1/2	1/2 −1/2	3/2 −3/2
+1 +1/2	1					
+1 −1/2		1/3	2/3			
0 +1/2		2/3	−1/3			
0 −1/2				2/3	1/3	
−1 +1/2				1/3	−2/3	
−1 −1/2						1

$l = 1 \left\{ \right.$

Enumerating the $l = 1$ degenerate states



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$$l = 1 \left\{ \psi_3 \equiv \left| \tfrac{3}{2}\, \tfrac{3}{2} \right\rangle \right.$$

Enumerating the $l = 1$ degenerate states

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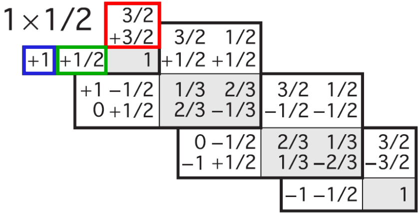

$$l = 1 \begin{cases} \psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle = |1 1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \\ \\ \end{cases}$$

Enumerating the $l = 1$ degenerate states

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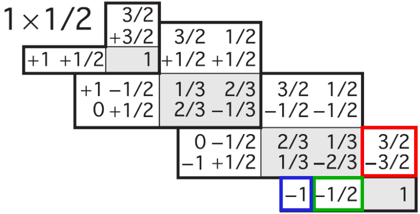

$$l = 1 \begin{cases} \psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle \qquad = |11\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \psi_4 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{3}{2}\right\rangle \end{cases}$$

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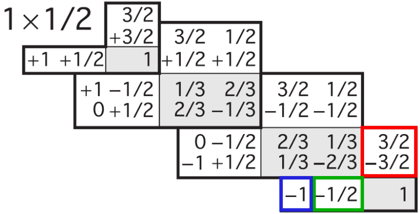

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$$

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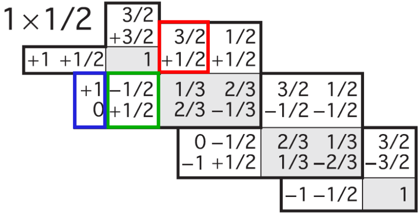

$$l = 1 \begin{cases} \psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle & = |1 1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \psi_4 \equiv \left|\tfrac{3}{2}\,{-\tfrac{3}{2}}\right\rangle & = |1\,{-1}\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \\ \psi_5 \equiv \left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle & \end{cases}$$

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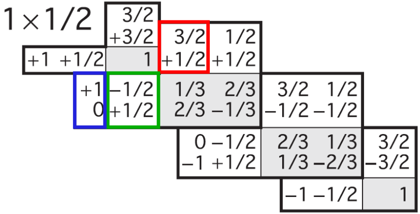

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\psi_5 \equiv \left|\frac{3}{2}\,\frac{1}{2}\right\rangle & = \sqrt{\frac{2}{3}}\,|10\rangle \left|\frac{1}{2}\,\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\,|11\rangle \left|\frac{1}{2}\,{-\frac{1}{2}}\right\rangle
\end{cases}
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Enumerating the $l = 1$ degenerate states

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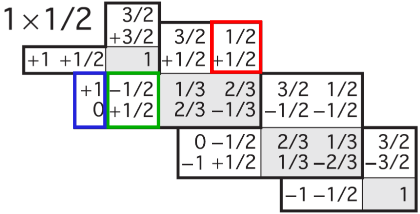

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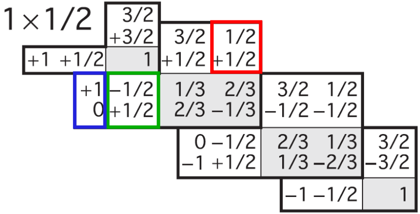
Enumerating the $l = 1$ degenerate states

Pick out all the possible combinations to build the remaining degenerate states



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l = 1 \begin{cases}
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\psi_5 \equiv \left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{2}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\,|11\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\
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\end{cases}
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Enumerating the $l = 1$ degenerate states

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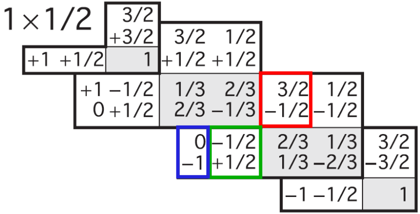

$$
l = 1 \begin{cases}
\psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle & = |1 1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\
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\psi_7 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{1}{2}\right\rangle &
\end{cases}
$$

Enumerating the $l = 1$ degenerate states

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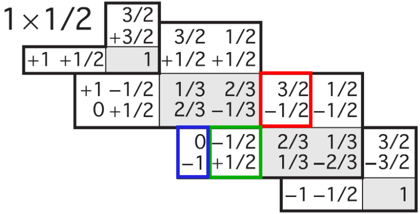

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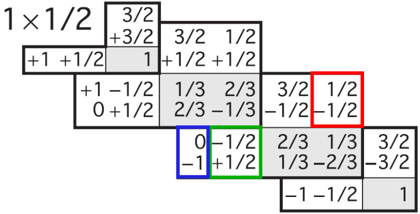
\psi_7 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{1}{3}}\,|1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle
\end{cases}
$$

Enumerating the $l = 1$ degenerate states



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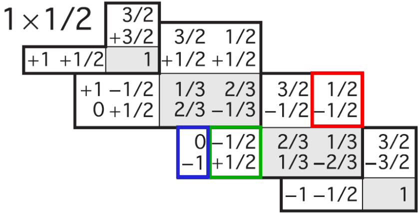
Enumerating the $l = 1$ degenerate states

Pick out all the possible combinations to build the remaining degenerate states



$$l = 1 \begin{cases} \psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle & = \left|1 1\right\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \psi_4 \equiv \left|\tfrac{3}{2}\,{-\tfrac{3}{2}}\right\rangle & = \left|1\, {-1}\right\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \\ \psi_5 \equiv \left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{2}{3}} \left|1 0\right\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}} \left|1 1\right\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \\ \psi_6 \equiv \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle & = -\sqrt{\tfrac{1}{3}} \left|1 0\right\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}} \left|1 1\right\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \\ \psi_7 \equiv \left|\tfrac{3}{2}\,{-\tfrac{1}{2}}\right\rangle & = \sqrt{\tfrac{1}{3}} \left|1\, {-1}\right\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}} \left|1 0\right\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \\ \psi_8 \equiv \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle & = -\sqrt{\tfrac{2}{3}} \left|1\, {-1}\right\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}} \left|1 0\right\rangle \left|\tfrac{1}{2}\,{-\tfrac{1}{2}}\right\rangle \end{cases}$$

Complete $n = 2$, $l = 0, 1$ degenerate set

The complete set of 8 $n = 2$, $l = 0, 1$ degenerate states using quantum numbers l, s, j, and m_j can now be listed

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$$
l = 0 \begin{cases}
\psi_1 \equiv \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle & = |00\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\
\psi_2 \equiv \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = |00\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle
\end{cases}
$$

$$
l = 1 \begin{cases}
\psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle & = |11\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\
\psi_4 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{3}{2}\right\rangle & = |1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\
\psi_5 \equiv \left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{2}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\,|11\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\
\psi_6 \equiv \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle & = -\sqrt{\tfrac{1}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\,|11\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\
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\psi_8 \equiv \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = -\sqrt{\tfrac{2}{3}}\,|1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle
\end{cases}
$$

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The complete set of 8 $n = 2$, $l = 0, 1$ degenerate states using quantum numbers l, s, j, and m_j can now be listed

$$
l = 0 \begin{cases} \psi_1 \equiv \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle & = |00\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \psi_2 \equiv \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = |00\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \end{cases}
$$

$$
l = 1 \begin{cases} \psi_3 \equiv \left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle & = |11\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle \\ \psi_4 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{3}{2}\right\rangle & = |1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\ \psi_5 \equiv \left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{2}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\,|11\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\ \psi_6 \equiv \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle & = -\sqrt{\tfrac{1}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\,|11\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\ \psi_7 \equiv \left|\tfrac{3}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = \sqrt{\tfrac{1}{3}}\,|1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \\ \psi_8 \equiv \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle & = -\sqrt{\tfrac{2}{3}}\,|1\,\text{-}1\rangle \left|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\,|10\rangle \left|\tfrac{1}{2}\,\text{-}\tfrac{1}{2}\right\rangle \end{cases}
$$

now build the W matrix for the $H' = H'_Z + H'_{fs}$ perturbation

The W-matrix

All the matrix elements in this basis set will be calculated by you when you do problem 7.29. Defining the fine structure and Zeeman terms with

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 & & \gamma-2\beta & & & & & \\
 & & & \gamma+2\beta & & & & \\
 & & & & \gamma-\frac{2}{3}\beta & \frac{\sqrt{2}}{3}\beta & & \\
 & & & & \frac{\sqrt{2}}{3}\beta & 5\gamma-\frac{1}{3}\beta & & \\
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ψ_1, ψ_2, ψ_3, and ψ_4 are clearly already eigenfunctions of the full perturbation. The other 4 eigenfunctions can be solved by solving two 2×2 blocks and then negating the solutions

Block 1 solution

The first block is solved by diagonalizing the 2×2 sub-matrix

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$$0 = det \begin{vmatrix} \gamma - \frac{2}{3}\beta - \lambda & \frac{\sqrt{2}}{3}\beta \\ \frac{\sqrt{2}}{3}\beta & 5\gamma - \frac{1}{3}\beta - \lambda \end{vmatrix}$$

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$$0 = (\gamma - \frac{2}{3}\beta - \lambda)(5\gamma - \frac{1}{3}\beta - \lambda) - \frac{2}{9}\beta^2$$

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$$\begin{aligned} 0 &= (\gamma - \frac{2}{3}\beta - \lambda)(5\gamma - \frac{1}{3}\beta - \lambda) - \frac{2}{9}\beta^2 \\ &= \lambda^2 + 5\gamma^2 + \frac{2}{9}\beta^2 - \frac{11}{3}\beta\gamma - 6\gamma\lambda + \beta\lambda - \frac{2}{9}\beta^2 \end{aligned}$$

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using the quadratic equation and recalling that the solution must be negated

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$$-\lambda_\pm = 3\gamma - \frac{\beta}{2} \pm \frac{1}{2}\sqrt{36\gamma^2 - 12\beta\gamma + \beta^2 - 20\gamma^2 + \frac{44}{3}\beta\gamma}$$

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$$\lambda_\pm = -3\gamma + \frac{\beta}{2} \pm \sqrt{4\gamma^2 + \frac{2}{3}\gamma\beta + \frac{1}{4}\beta^2}$$

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$$0 = det\begin{vmatrix} \gamma+\frac{2}{3}\beta-\lambda & \frac{\sqrt{2}}{3}\beta \\ \frac{\sqrt{2}}{3}\beta & 5\gamma+\frac{1}{3}\beta-\lambda \end{vmatrix}$$

$$\begin{aligned} 0 &= (\gamma+\frac{2}{3}\beta-\lambda)(5\gamma+\frac{1}{3}\beta-\lambda)-\frac{2}{9}\beta^2 \\ &= \lambda^2+5\gamma^2+\frac{2}{9}\beta^2+\frac{11}{3}\beta\gamma-6\gamma\lambda-\beta\lambda-\frac{2}{9}\beta^2 \end{aligned}$$

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$$0 = det \begin{vmatrix} \gamma + \frac{2}{3}\beta - \lambda & \frac{\sqrt{2}}{3}\beta \\ \frac{\sqrt{2}}{3}\beta & 5\gamma + \frac{1}{3}\beta - \lambda \end{vmatrix}$$

$$\begin{aligned} 0 &= (\gamma + \frac{2}{3}\beta - \lambda)(5\gamma + \frac{1}{3}\beta - \lambda) - \frac{2}{9}\beta^2 \\ &= \lambda^2 + 5\gamma^2 + \cancel{\frac{2}{9}\beta^2} + \frac{11}{3}\beta\gamma - 6\gamma\lambda - \beta\lambda - \cancel{\frac{2}{9}\beta^2} \\ &= \lambda^2 - \lambda(6\gamma + \beta) + \left(5\gamma^2 + \frac{11}{3}\gamma\beta\right) \end{aligned}$$

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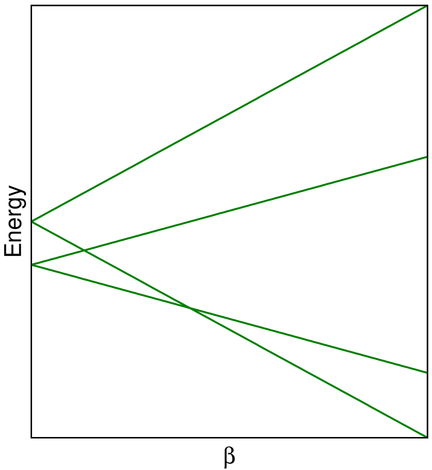
$$\lambda_\pm = -3\gamma - \frac{\beta}{2} \pm \sqrt{4\gamma^2 - \frac{2}{3}\gamma\beta + \frac{1}{4}\beta^2}$$

These are completely general solutions which hold for all values of the magnetic field strength

Zeeman effect eigenvalues



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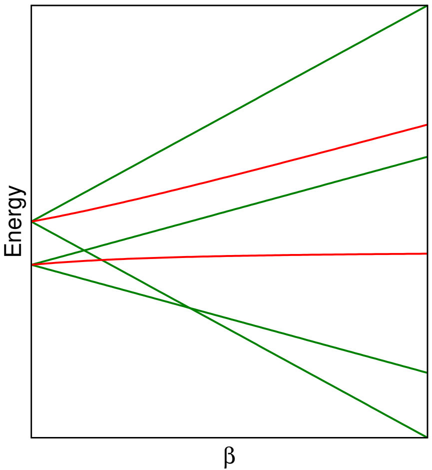

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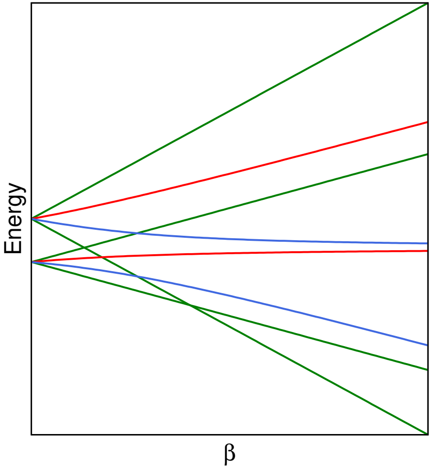

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Problem 7.31

Let $\vec{a}$ and $\vec{b}$ be two constant vectors. Show that

$$\int (\vec{a} \cdot \hat{r})(\vec{b} \cdot \hat{r}) \sin\theta \, d\theta \, d\phi = \frac{4\pi}{3}(\vec{a} \cdot \vec{b})$$

Use this result to demonstrate that

$$\left\langle \frac{3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e}{r^3} \right\rangle = 0$$

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$$I \equiv \int (a_x\sin\theta\cos\phi + a_y\sin\theta\sin\phi + a_z\cos\theta)\cdot$$
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$$I = \frac{4}{3}\pi(a_x b_x + a_y b_y) + \frac{2}{3}2\pi a_z b_z$$

Problem 7.31 (cont.)

$$I \equiv \int (a_x \sin\theta\cos\phi + a_y \sin\theta\sin\phi + a_z\cos\theta)\cdot$$
$$(b_x \sin\theta\cos\phi + b_y \sin\theta\sin\phi + b_z\cos\theta)\sin\theta\, d\theta\, d\phi$$

$$0 = \int_0^{2\pi} \sin\phi\, d\phi = \int_0^{2\pi} \cos\phi\, d\phi = \int_0^{2\pi} \sin\phi\cos\phi\, d\phi$$

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$$\int_0^{2\pi} \sin^2\phi\, d\phi = \pi = \int_0^{2\pi} \cos^2\phi\, d\phi, \qquad \int_0^{2\pi} d\phi = 2\pi$$

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Problem 7.31 (cont.)

For states with $l = 0$, the angular wave function is independent of θ and ϕ (e.g. $Y_0^0 = 1/\sqrt{4\pi}$) so we can separate the radial and angular integral

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A and B cancel exactly, giving the desired result

Spin-spin coupling

$$E_{hf}^{(1)} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e} \left\langle \frac{3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e}{r^3} \right\rangle + \frac{\mu_0 g_p e^2}{3 m_p m_e} \langle \vec{S}_p \cdot \vec{S}_e \rangle |\psi(0)|^2$$

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both the proton and the electron have spin $1/2$

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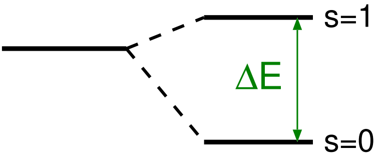

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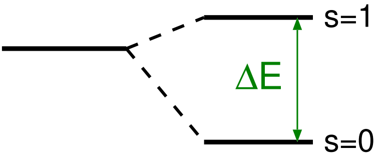
$$\Delta E = 5.88 \times 10^{-6}\text{eV}$$



The 21 cm hydrogen line

$$E_{hf}^{(1)} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\langle \vec{S}_p \cdot \vec{S}_e\rangle$$

$$= \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\frac{\hbar^2}{2}[s(s+1) - s_e(s_e+1) - s_p(s_p+1)]$$

both the proton and the electron have spin $1/2$

$$s_e(s_e+1) = s_p(s_p+1) = \frac{3}{4}$$

but the total spin can be in either a singlet or a triplet state

$$s = 0 \quad \rightarrow \quad s(s+1) = 0$$
$$s = 1 \quad \rightarrow \quad s(s+1) = 2$$

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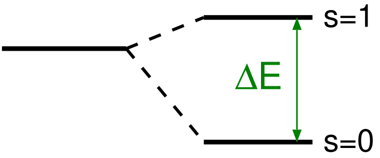

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