Today's Outline - January 28, 2013

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- First order PT examples

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- Second order PT

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- Second order PT
- Degenerate PT

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Reading Assignment: Chapter 6.3

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Homework Assignment #02:
Chapter 5: 27, 30; Chapter 6: 1, 4, 6, 29
due Monday, February 4, 2013

First order perturbation theory review

Unperturbed Hamiltonian, H^0
with solutions ψ_n^0

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"perturbed" Hamiltonian,

$$H = H^0 + H'$$

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$$\psi_n^1 = \sum_{m\neq n} \frac{\langle\psi_m^0 \left| H' \right| \psi_n^0\rangle}{(E_n^0 - E_m^0)}\psi_m^0$$

Example 6.1

Constant potential, V_0 in an infinite square well from $x = 0$ to $x = a$

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this is not an exact solution but the first term in a series of energy correction terms