Today's Outline - January 16, 2013

Today's Outline - January 16, 2013

- Band theory (cont.)

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- Quantum statistics

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Reading Assignment: Chapter 6.1–6.2

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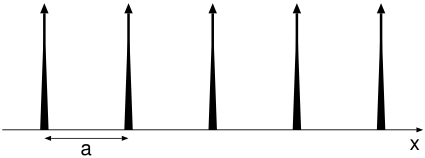
Reading Assignment: Chapter 6.1–6.2

Homework Assignment #01:
Chapter 5: 18, 20, 21, 23, 24, 26
due Wednesday, January 23, 2013

Dirac comb problem

Solve the Dirac comb potential using Bloch's theorem and periodic boundary conditions by solving over a single unit



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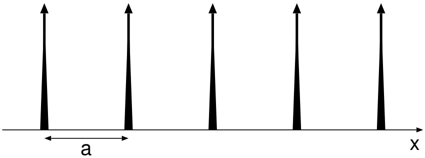

$$\psi(x) = \psi(x + Na)$$

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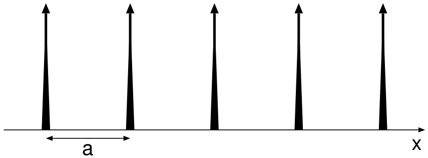

$$\psi(x) = \psi(x + Na) = e^{iNKa}\psi(x)$$

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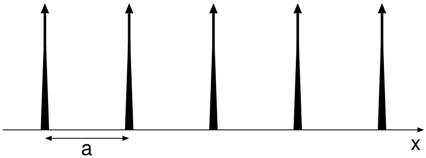


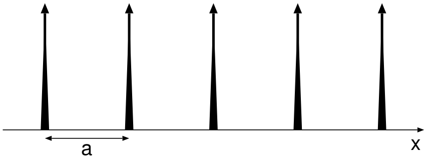
$$\psi(x) = \psi(x + Na) = e^{iNKa}\psi(x)$$

$$K = \frac{2\pi n}{Na}, \quad (n = 0, \pm 1, \pm 2, \ldots)$$

Dirac comb problem

Solve the Dirac comb potential using Bloch's theorem and periodic boundary conditions by solving over a single unit

the potential can be written as a sum of delta functions



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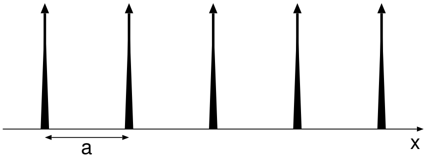

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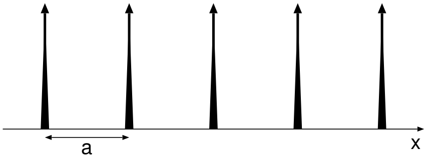
$$K = \frac{2\pi n}{Na}, \quad (n = 0, \pm 1, \pm 2, \dots)$$

$$V(x) = \alpha \sum_{j=0}^{N-1} \delta(x - ja)$$

Dirac comb problem

Solve the Dirac comb potential using Bloch's theorem and periodic boundary conditions by solving over a single unit

the potential can be written as a sum of delta functions, where α is the "strength" of the potentials and N is the number of repeating periods



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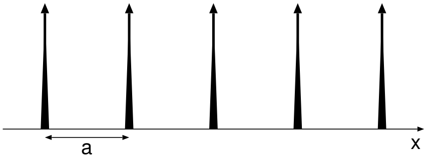

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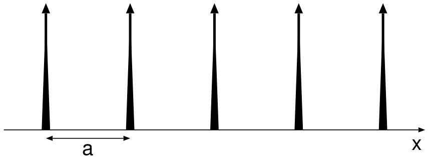

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$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$$

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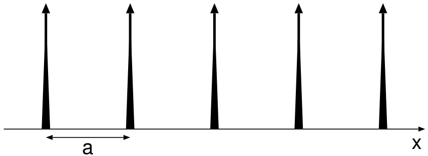
in the region $0 < x < a$ we have defining $k = \sqrt{2mE}/\hbar$, gives



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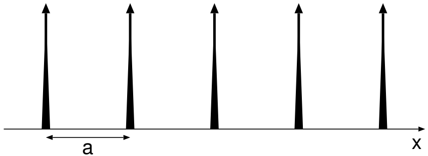

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defining $k = \sqrt{2mE}/\hbar$, gives

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

Dirac comb solution

The general solution is one we have seen already

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$$\psi(x) = A\sin(kx) + B\cos(kx), \quad (0 < x < a)$$

The general solution is one we have seen already and in the cell immediately preceeding the origin, we can write the solution using Bloch's theorem

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$$\psi(x) = A\sin(kx) + B\cos(kx), \quad (0 < x < a)$$
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rearranging the continuity equation, substituting for A, and cancelling B

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$$[e^{iKa}-\cos(ka)][1-e^{-iKa}\cos(ka)] + e^{-iKa}\sin^2(ka) = \frac{2m\alpha}{\hbar^2 k}\sin(ka)$$

Dirac comb solution

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$$e^{iKa}-\cos(ka)-\cos(ka)+e^{-iKa}\cos^2(ka) \qquad = \frac{2m\alpha}{\hbar^2 k}\sin(ka)$$

Dirac comb solution

$$k\frac{[e^{iKa}-\cos(ka)]}{\sin(ka)} - e^{-iKa}k\frac{[e^{iKa}-\cos(ka)]}{\sin(ka)}\cos(ka) + e^{-iKa}k\sin(ka) = \frac{2m\alpha}{\hbar^2}$$

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Dirac comb solution

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$$e^{iKa} - 2\cos(ka) + e^{-iKa} = \frac{2m\alpha}{\hbar^2 k}\sin(ka)$$

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Dirac comb solution

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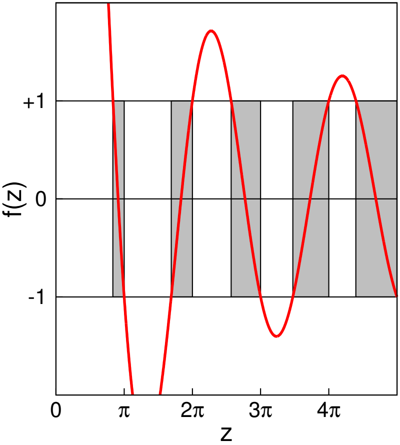

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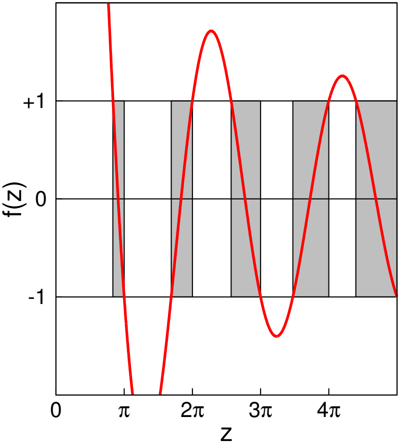

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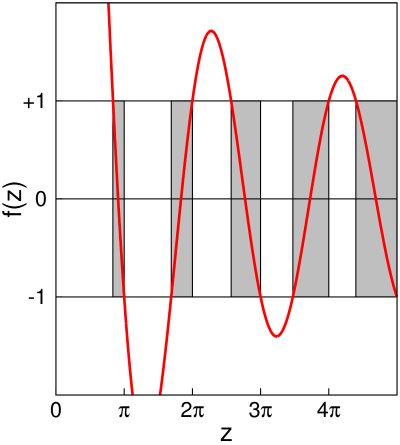

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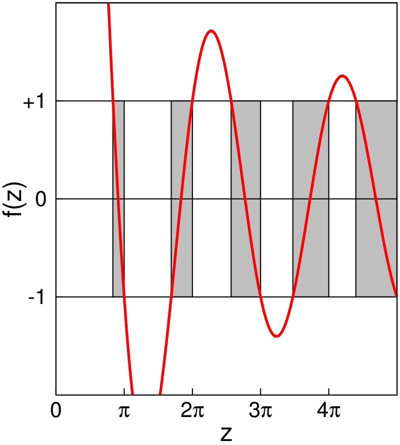

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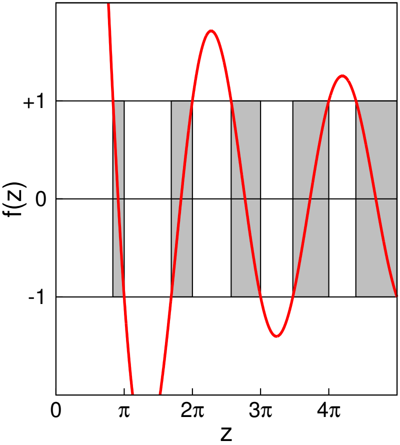

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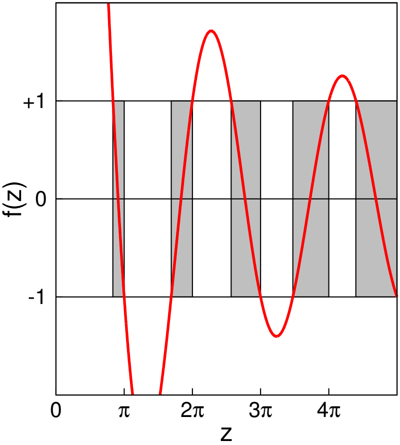

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the bands and gaps are able to explain the properties of metals, semiconductors and insulators in a simple way

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How does quantum mechanics affect our development of statistical mechanics?

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\begin{array}{ccc}
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