The statistical interpretation

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- The meaning of the wavefunction

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- Probability review

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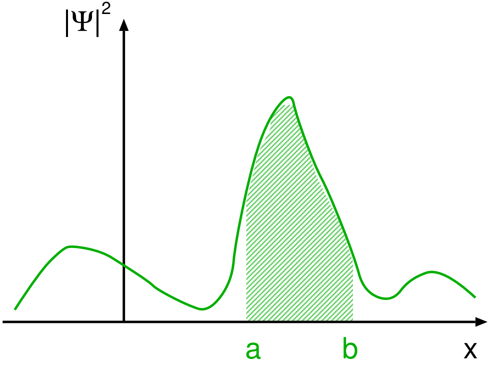

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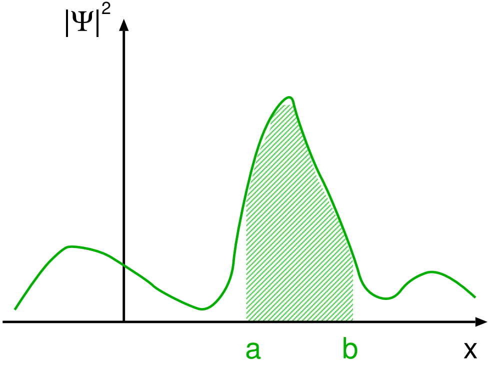

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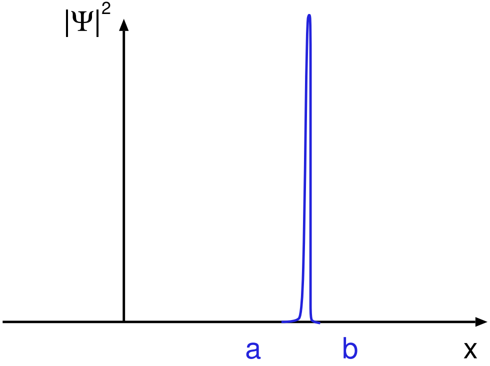

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Since $\sigma^2 \geq 0$, $\langle j^2\rangle \geq \langle j\rangle^2$

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Normalizing the wave function

Beyond satisfying the Schrödinger equation, the wave function must also have physical significance.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

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Beyond satisfying the Schrödinger equation, the wave function must also have physical significance. If we are to believe the statistical interpretation, the wave-function must be *normalized*, that is the integral of the probability density over all space must be unity.

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$$\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar}V\Psi$$

and its complex conjugate

$$\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar}V\Psi^*$$

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Thus the wave function remains normalized at all times $t \geq 0$.

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This will eventually lead directly to what is called the "bra-ket" notation commonly used in quantum mechanics.

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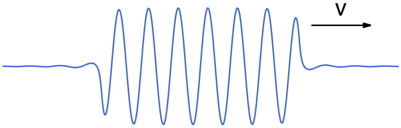

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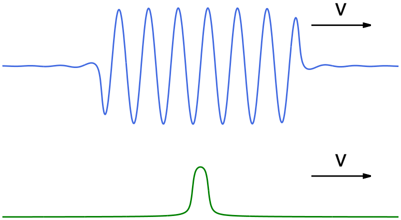

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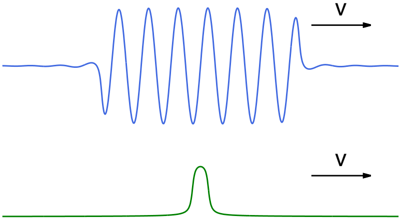

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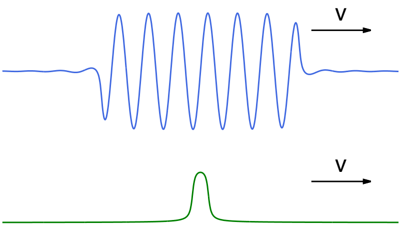

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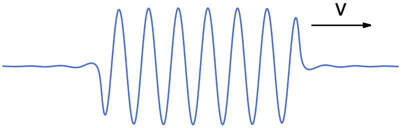

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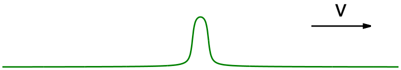

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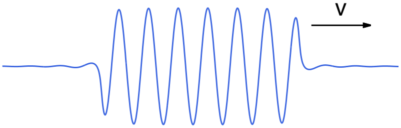

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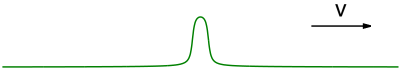

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