A short review of modern physics

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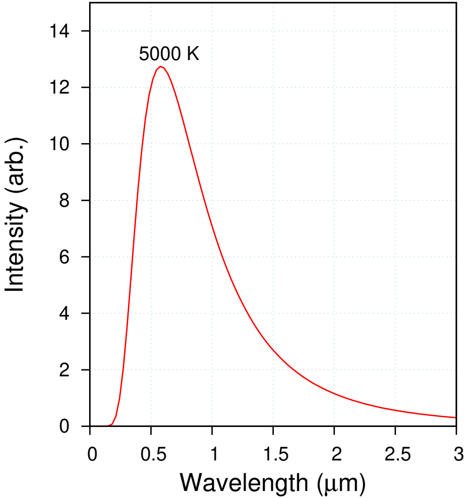

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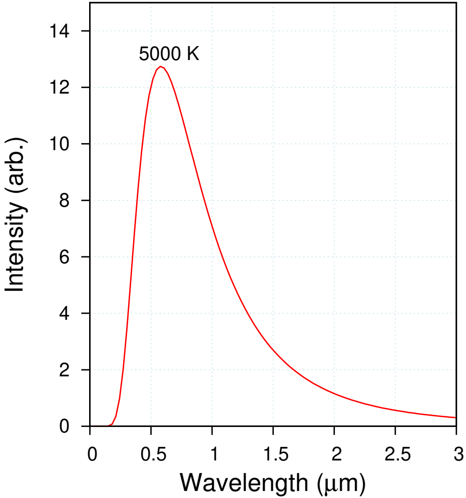

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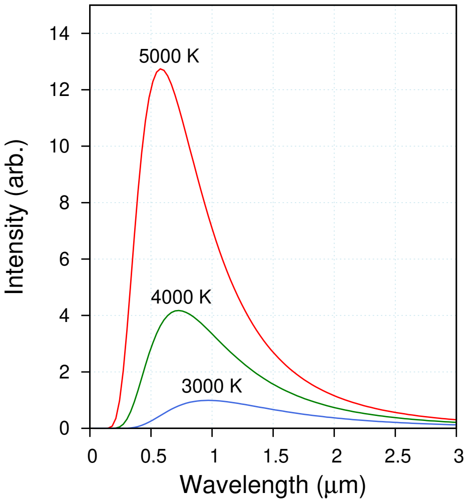

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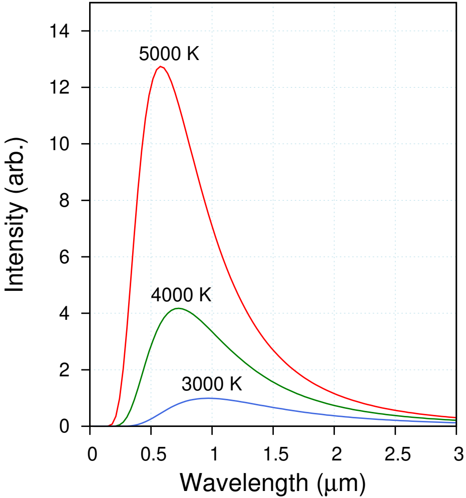

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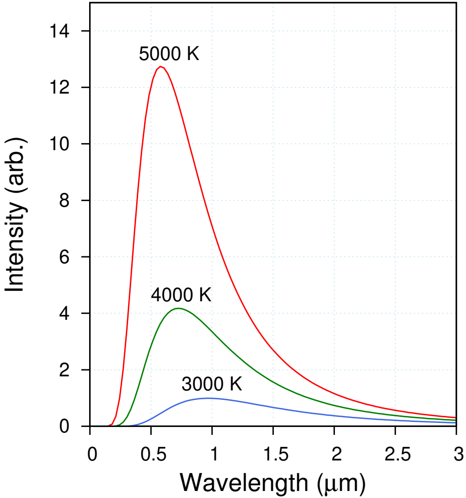

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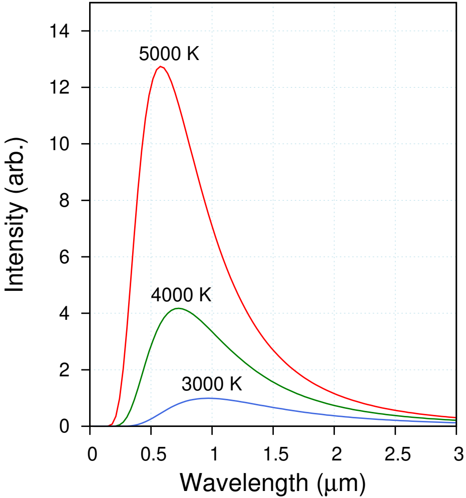

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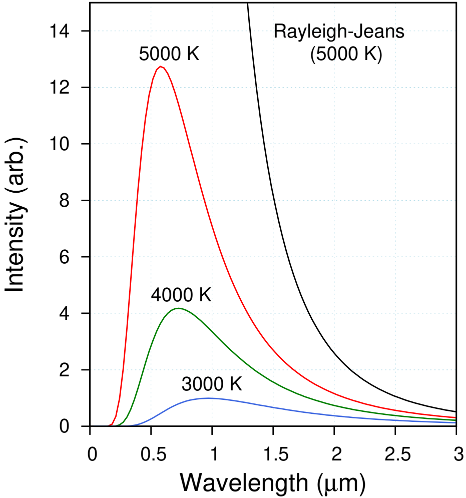
$$\int_0^\infty u(\lambda) d\lambda \propto \int_0^\infty \lambda^{-4} d\lambda \rightarrow \infty$$



Planck's solution

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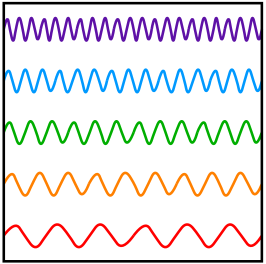

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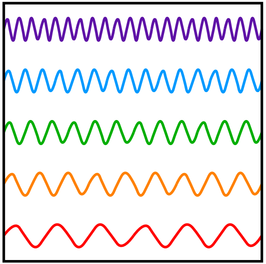

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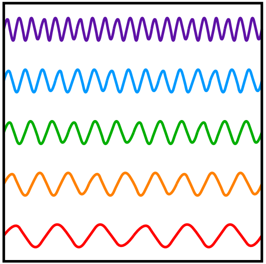

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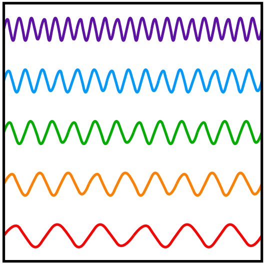

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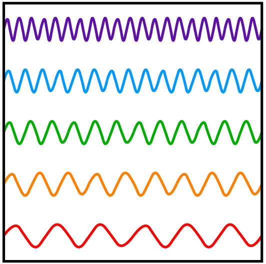

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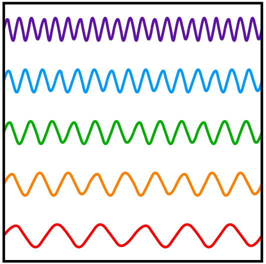

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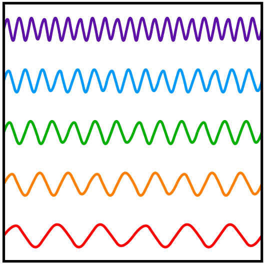

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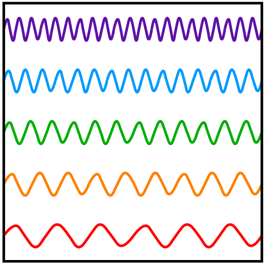

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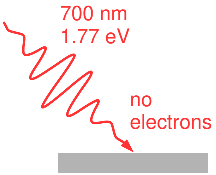

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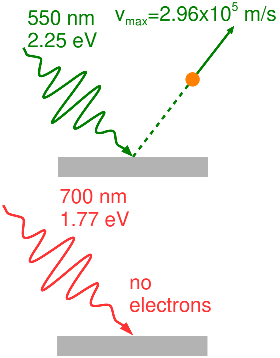

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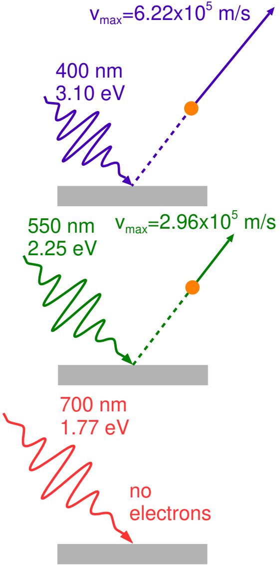
the maximum velocity of the emitted electrons measured by the stopping potential increases



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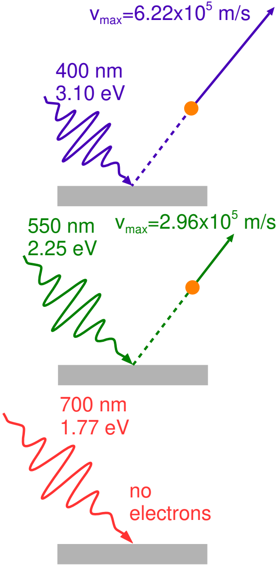

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the threshold wavelength is directly related to the work function, ϕ, of the metal

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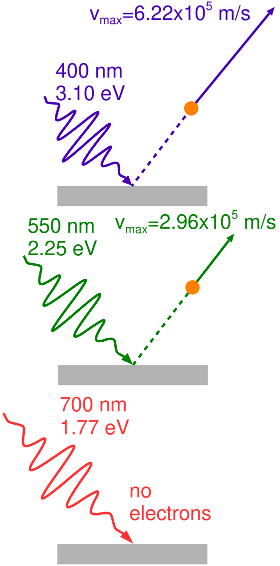

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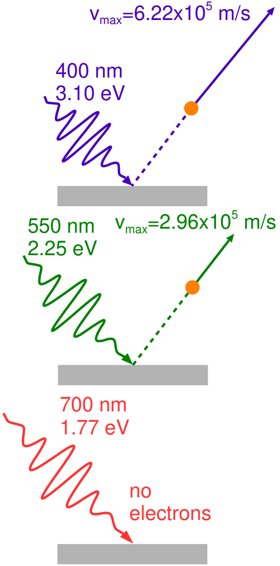

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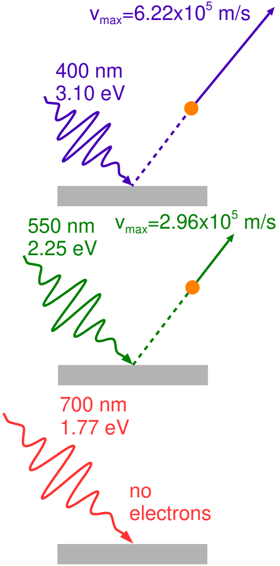

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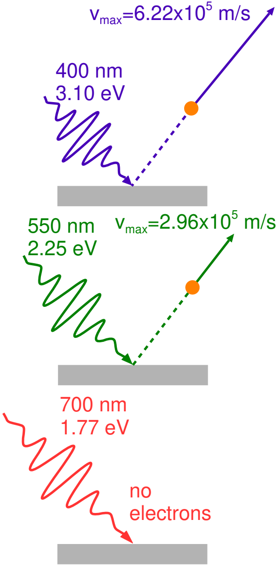
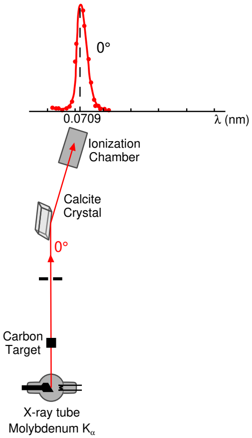
the intensity of the light, not the energy of the photon, determines how many electrons are emitted



Compton scattering experiment



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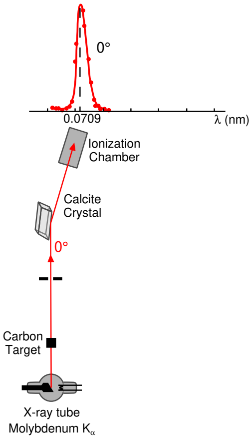

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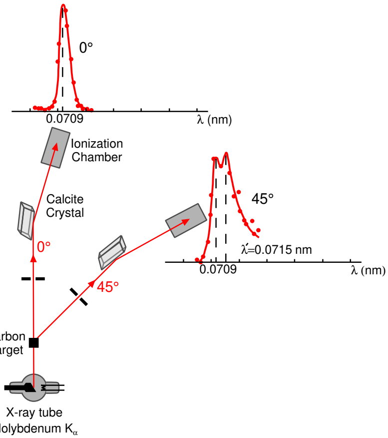

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as the scattering angle was increased, x-rays at longer wavelengths (lower energies) than the incident energy were observed

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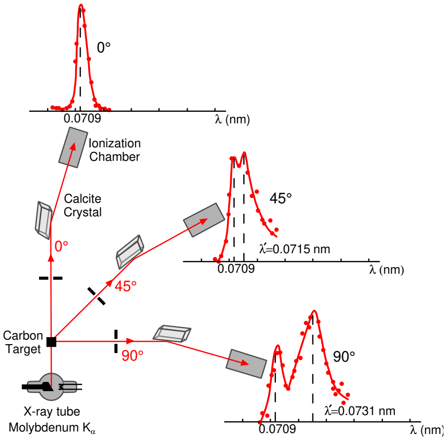

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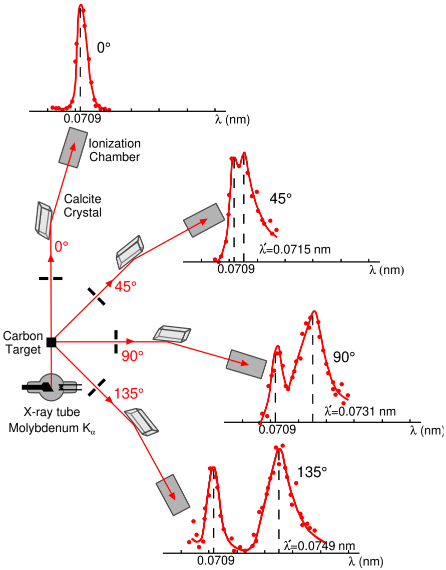

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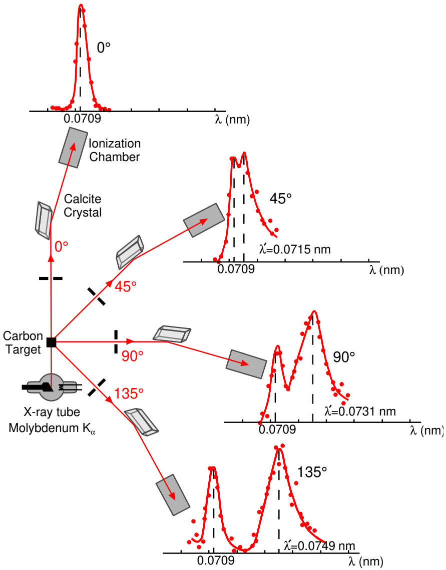

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the peak wavelength of the Compton scattering peak can be calculated by applying relativistic scattering theory

Compton scattering phenomenon

A photon-electron collision

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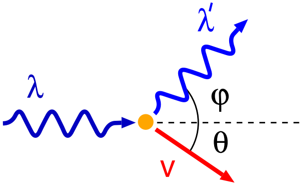

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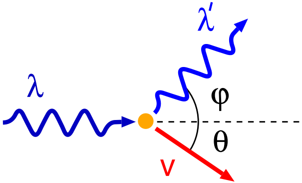

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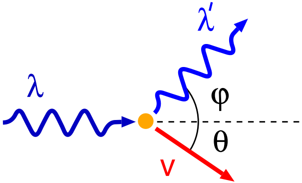

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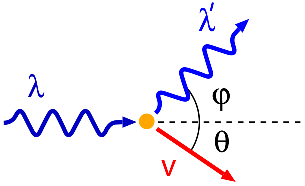

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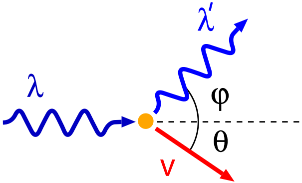

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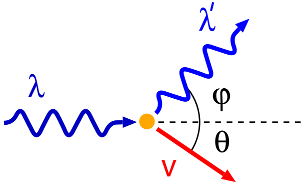

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$$mc^2 + \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \gamma mc^2 \quad \text{(energy)}$$

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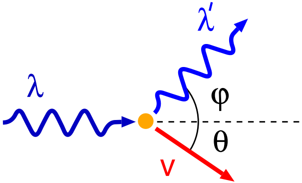

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$$mc^2 + \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \gamma mc^2 \quad \text{(energy)}$$

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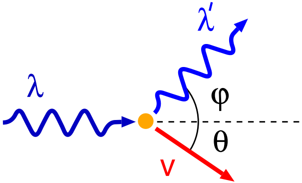

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Compton scattering derivation

squaring the momentum equations

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squaring the momentum equations

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now add them together,

$$\gamma^2 m^2 v^2 \left(\sin^2\theta + \cos^2\theta\right) = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 + \left(-\frac{h}{\lambda'}\sin\varphi\right)^2$$

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squaring the momentum equations

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now add them together, substitute $\sin^2\theta + \cos^2\theta = 1$, expand the squares,

$$\gamma^2 m^2 v^2 \left(\sin^2\theta + \cos^2\theta\right) = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 + \left(-\frac{h}{\lambda'}\sin\varphi\right)^2$$

$$\gamma^2 m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \frac{h^2}{\lambda'^2}\sin^2\varphi + \frac{h^2}{\lambda'^2}\cos^2\varphi$$

Compton scattering derivation

squaring the momentum equations

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 = \gamma^2 m^2 v^2 \cos^2\theta$$

$$\left(-\frac{h}{\lambda'}\sin\varphi\right)^2 = \gamma^2 m^2 v^2 \sin^2\theta$$

now add them together, substitute $\sin^2\theta + \cos^2\theta = 1$, expand the squares, and $\sin^2\varphi + \cos^2\varphi = 1$, then rearrange

$$\gamma^2 m^2 v^2 \left(\sin^2\theta + \cos^2\theta\right) = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 + \left(-\frac{h}{\lambda'}\sin\varphi\right)^2$$

$$\gamma^2 m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \frac{h^2}{\lambda'^2}\sin^2\varphi + \frac{h^2}{\lambda'^2}\cos^2\varphi$$

$$\frac{m^2 v^2}{1-\beta^2} = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \frac{h^2}{\lambda'^2}$$

Compton scattering derivation

squaring the momentum equations

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 = \gamma^2 m^2 v^2 \cos^2\theta$$

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now add them together, substitute $\sin^2\theta + \cos^2\theta = 1$, expand the squares, and $\sin^2\varphi + \cos^2\varphi = 1$, then rearrange and substitute $v = \beta c$

$$\gamma^2 m^2 v^2 \left(\sin^2\theta + \cos^2\theta\right) = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 + \left(-\frac{h}{\lambda'}\sin\varphi\right)^2$$

$$\gamma^2 m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \frac{h^2}{\lambda'^2}\sin^2\varphi + \frac{h^2}{\lambda'^2}\cos^2\varphi$$

$$\frac{m^2 c^2 \beta^2}{1-\beta^2} = \frac{m^2 v^2}{1-\beta^2} = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \frac{h^2}{\lambda'^2}$$

Compton scattering derivation (cont.)

Now take the energy equation and square it,

$$\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1-\beta^2}$$

Compton scattering derivation (cont.)

Now take the energy equation and square it, then solve it for β^2

$$\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1-\beta^2}$$

$$\beta^2 = 1 - \frac{m^2 c^4}{\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2}$$

Compton scattering derivation (cont.)

Now take the energy equation and square it, then solve it for β^2 which is substituted into the equation from the momentum.

$$\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1-\beta^2}$$

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$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \frac{m^2 c^2 \beta^2}{1-\beta^2}$$

Compton scattering derivation (cont.)

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$$\begin{aligned}\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi &= \frac{m^2 c^2 \beta^2}{1-\beta^2} \\ &= \frac{1}{c^2}\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 - m^2 c^2\end{aligned}$$

Compton scattering derivation (cont.)

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2$$

Compton scattering derivation (cont.)

After expansion,

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2$$
$$= m^2c^2 + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda\lambda'} - m^2c^2$$

Compton scattering derivation (cont.)

After expansion, cancellation,

$$\begin{aligned}
\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi &= \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2 \\
&= \cancel{m^2c^2} + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda\lambda'} - \cancel{m^2c^2} \\
&= 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}
\end{aligned}$$

Compton scattering derivation (cont.)

After expansion, cancellation,

$$
\begin{aligned}
\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi &= \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2 \\
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&= 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}
\end{aligned}
$$

Compton scattering derivation (cont.)

After expansion, cancellation, and rearrangement, we obtain

$$
\begin{aligned}
\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi &= \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2 \\
&= \cancel{m^2c^2} + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda\lambda'} - \cancel{m^2c^2} \\
&= 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}
\end{aligned}
$$

$$
\frac{2h^2}{\lambda\lambda'}(1-\cos\varphi) = 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)
$$

Compton scattering derivation (cont.)

After expansion, cancellation, and rearrangement, we obtain

$$\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2$$

$$= \cancel{m^2c^2} + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda\lambda'} - \cancel{m^2c^2}$$

$$= 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}$$

$$\frac{2h^2}{\lambda\lambda'}(1-\cos\varphi) = 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) = 2mhc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right)$$

Compton scattering derivation (cont.)

After expansion, cancellation, and rearrangement, we obtain

$$\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2$$

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Compton scattering derivation (cont.)

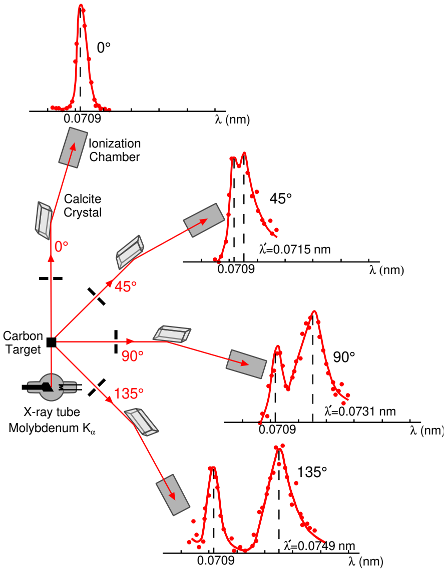
After expansion, cancellation, and rearrangement, we obtain

$$
\begin{aligned}
\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi &= \left(mc + \frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - m^2c^2 \\
&= \cancel{m^2c^2} + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda\lambda'} - \cancel{m^2c^2} \\
&= 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) + \cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}
\end{aligned}
$$

$$
\frac{2h^2}{\cancel{\lambda\lambda'}}(1-\cos\varphi) = 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) = 2mhc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{2mhc\Delta\lambda}{\cancel{\lambda\lambda'}}
$$

$$
\Delta\lambda = \frac{h}{mc}(1-\cos\varphi)
$$

Compton scattering equation



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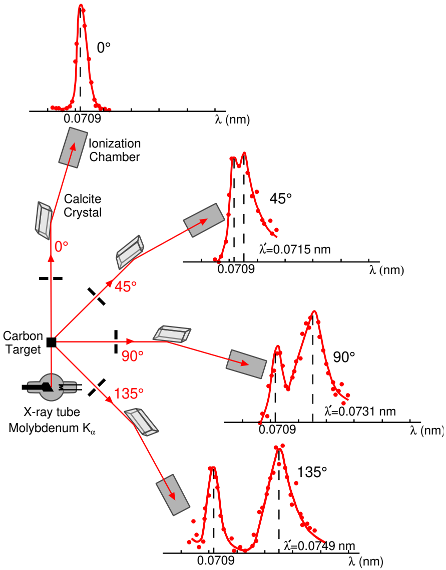
Compton scattering equation



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This explains the change in energy of the broader peak with increasing angle

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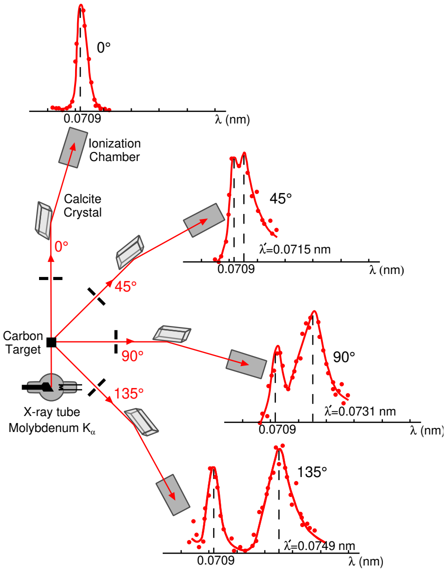

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it also provides the basis for understanding why the Compton peak is so broad . . .

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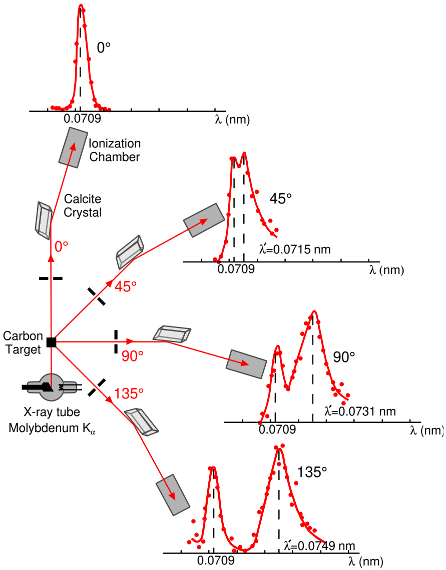

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This explains the change in energy of the broader peak with increasing angle

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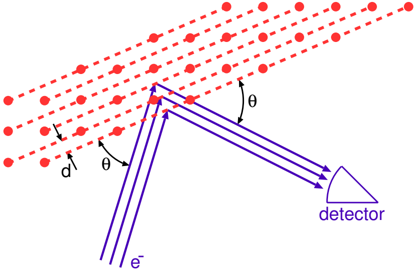
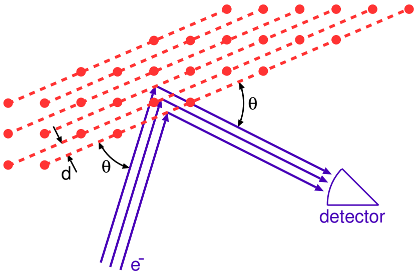
. . . since the electrons are not really "stationary", there will be a spread in energy and momentum of the outgoing photon.

Davisson-Germer experiment



In 1928, Davisson & Germer measured peaks in the reflectivity of electrons from a single crystal of nickel

Davisson-Germer experiment



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electrons were accelerated at varying potentials and incident on the nickel surface at a fixed angle, θ with a detector at 2θ from the incident beam

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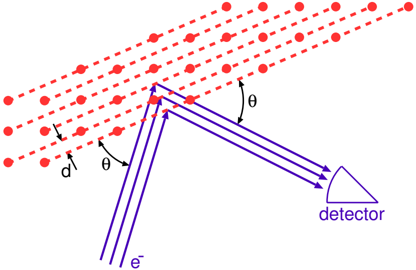

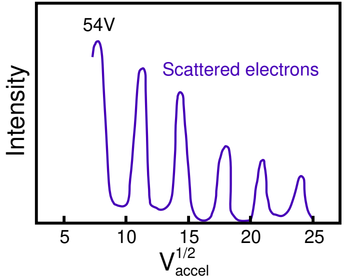

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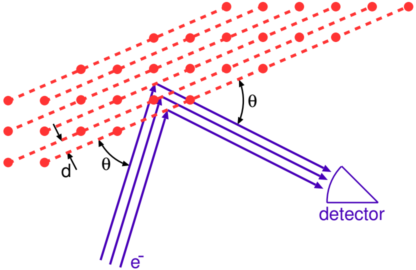

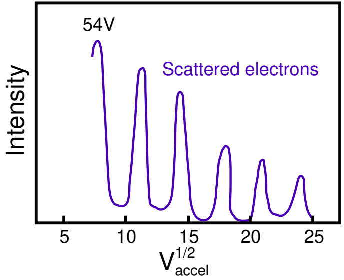

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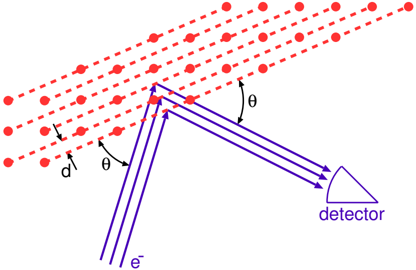
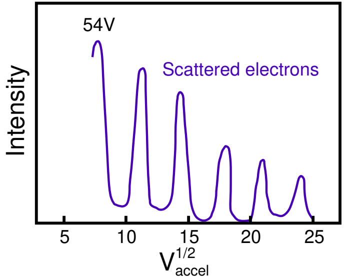
$$2d \sin\theta = n\lambda$$

Davisson-Germer experiment





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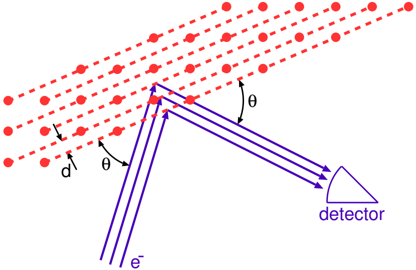

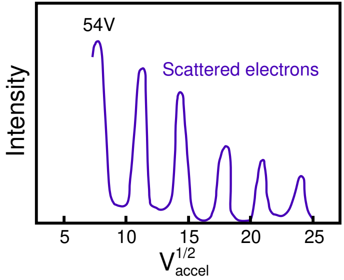

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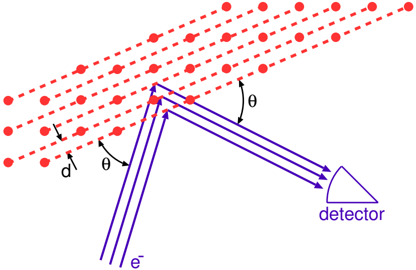

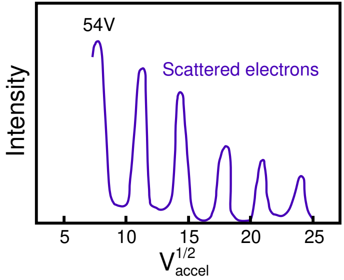

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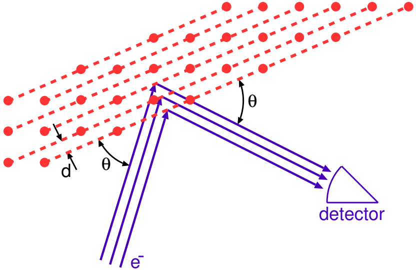

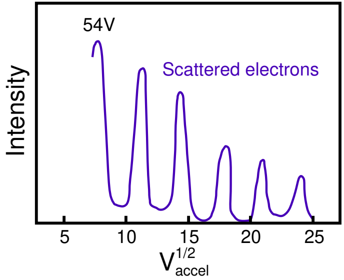

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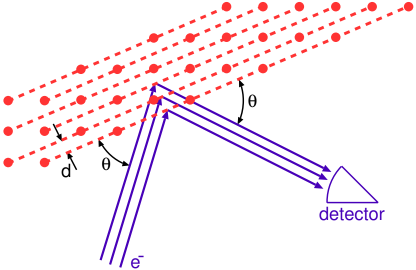

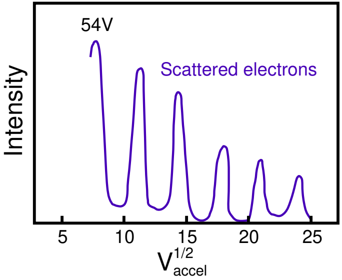

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the Davisson-Germer paper was published in the Proceedings of the National Academy of Sciences in 1928 and an historical account of the discovery was published in 1978 in Physics Today

The Schrödinger equation

The Schrödinger equation

- The 1-D Schrödinger equation

The Schrödinger equation

- The 1-D Schrödinger equation
- Deriving the Schrödinger equation

1D Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

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Schrödinger's wave mechanics was not the only formulation of quantum mechanics that was developed in that period

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$$V\Psi = \text{Potential Energy}$$

matrix mechanics was worked out in parallel by Heisenberg, Born, and Pascual and relativistic approaches were developed by Dirac and Pauli

Deriving the Schrödinger equation

"When Schrödinger first wrote it down, he gave a kind of derivation based on some heuristic arguments and some brilliant intuitive guesses. Some of the arguments he used were even false, but that does not matter; the only important thing is that the ultimate equation gives a correct description of nature." - Richard Feynman

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$$0 = \frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

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Inspired by wave optics, Schrödinger started with the wave equation for electromagnetic radiation with solution

$$E(x,t) = E_0 e^{i(kx-\omega t)}$$

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Deriving the Schrödinger equation

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