Hilbert space representations

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- Alternative bases

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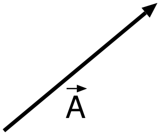

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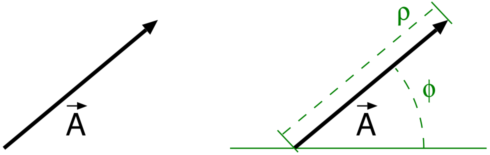

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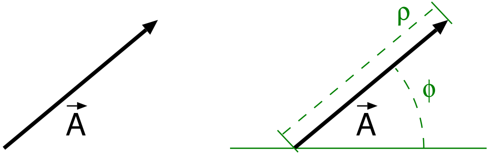

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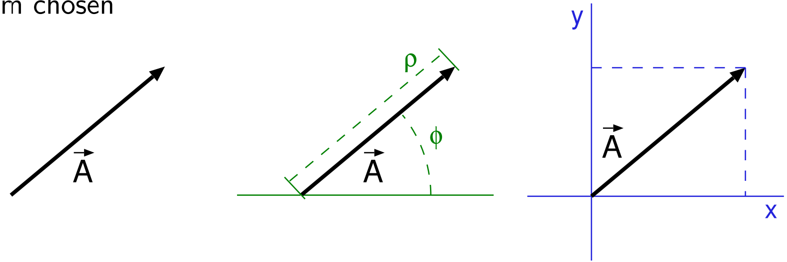

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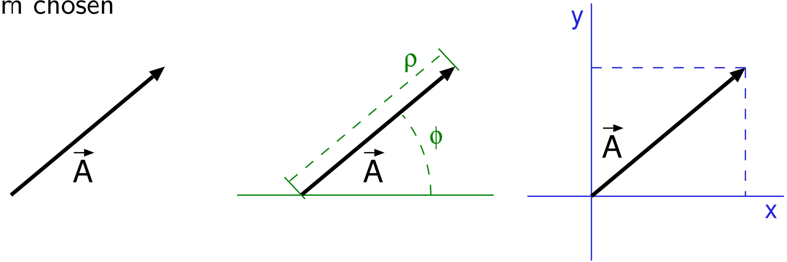

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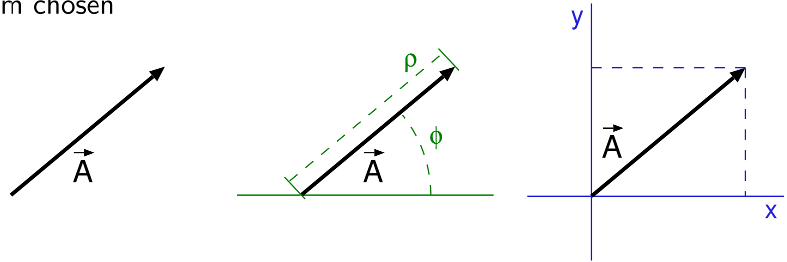

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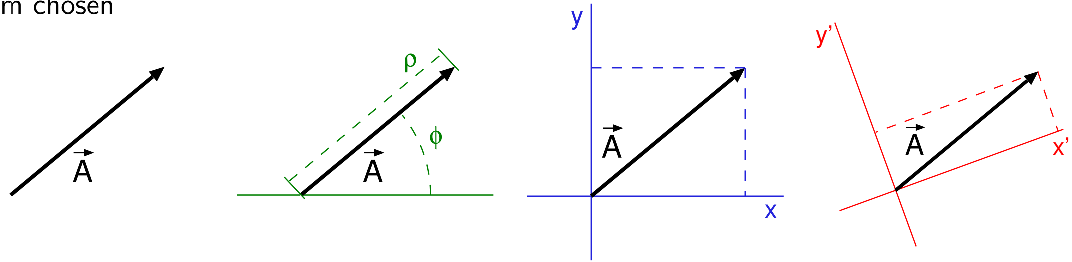

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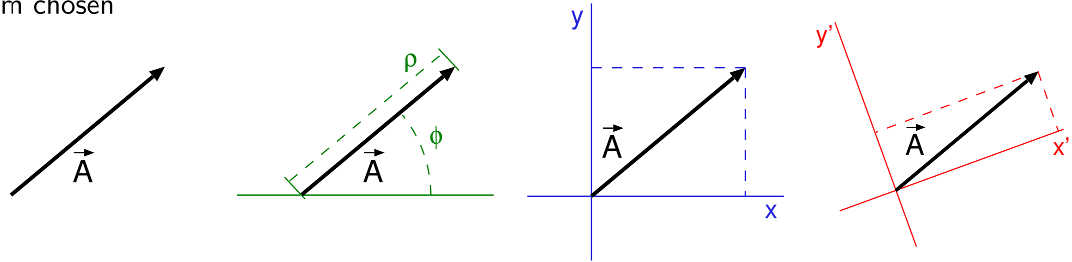

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this is simply the definition of a matrix applied to a vector

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this is convenient and will continue to be used but it is important to remember that it is just one possible representation of the state of the system in Hilbert space

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- Review of “bra-ket” notation

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there are more subtle aspects of this notation which we will discuss now

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Dirac notation allows us to abstract the action of the operator and then express it in any basis desired by taking an inner product with the appropriate basis set

consider the operation of $\hat{x}$ on the state *ket* $|\mathcal{S}(t)\rangle$

in the position basis, it is expressed as the action of the position operator in the x basis

$$\langle x|\hat{x}|\mathcal{S}(t)\rangle = x\Psi(x,t)$$

while in the momentum basis, it is the action of the position operator in the p basis

$$\langle p|\hat{x}|\mathcal{S}(t)\rangle = i\hbar \frac{\partial \Phi(p,t)}{\partial p}$$